Route 81 Conference 2016
Syracuse University

8:30 - 9:00	*Breakfast*	12:00 - 1:30	*Lunch*
9:00 - 9:10	*Special remarks*	1:30 - 2:00	**Ben Briggs**
9:10 - 9:40	**Nathan Grieve**	2:10 - 2:40	**Patricia Klein**
9:50 - 10:20	**Jack Huizenga**	2:40 - 3:10	*Coffee*
10:20 - 10:50	*Coffee*	3:10 - 3:40	**Rebecca R.G.**
10:50 - 11:20	**Josh Stangle**	3:50 - 4:20	**Luigi Ferraro**
11:30 - 12:00	**Vincent Gelinas**	4:30 - 5:00	**Kuei-Nuan Lin**

Dinner at Phoebe's Restaurant 6pm

Ben Briggs (University of Toronto)

Hochschild Cohomology and Koszul Duality

This is joint work with Vincent Gelinas. I will start by explaining some aspects of Koszul duality. In short, this concerns the relation between an augmented algebra A and its Yoneda algebra $\mathrm{Ext}_A(k,k)$. From the Hochschild cohomology $HH^*(A,A)$ there is a projection map to A, whose image is the centre of A, and a shearing map to $\mathrm{Ext}_A(k,k)$, whose image is more mysterious. Buchweitz, Green, Snashall and Soldberg have proven that the image is the graded centre of $\mathrm{Ext}_A(k,k)$ when A is Koszul. Actually, a pleasing symmetry between the projection and shearing maps under Koszul duality allows us to identify the image of the shearing map in general in terms of the higher structure on $\mathrm{Ext}_A(k,k)$: it is a kind of A_∞-centre. I will illustrate these ideas from a commutative algebra perspective. For instance, when A is a complete intersection, the Yoneda algebra contains the famous cohomology operators of Gulliksen, which live in the image of the shearing map. In this case we can compute the higher structure on $\mathrm{Ext}_A(k,k)$ explicitly. We may also talk about how this relates to the BGG correspondence, and to small, functorial resolutions over commutative algebras.

Luigi Ferraro (University of Nebraska)

Two classes of modules with infinite regularity

Let R be the graded ring $k[x_1,\ldots,x_n]/I$ where I is a homogeneous ideal of the polynomial ring with the standard grading. If the Castelnuovo-Mumford regularity of k is infinite we are going to construct a class of modules with infinite regularity. If R is a complete intersection we will be able to construct a second class of modules with infinite regularity.

Vincent Gelinas (University of Toronto)

Models for Maximal Cohen-Macaulay modules

The stable category of MCM modules over Gorenstein rings sits at the intersection of commutative algebra, singularity theory and representation theory, and their study exhibits the rich interplay between these fields. We will discuss explicit models for MCM modules through the simplest of examples.

Nathan Grieve (University of New Brunswick)

On Kodaira Dimension of Maximal Orders

This is a report on joint work in progress with Colin Ingalls. Specifically, I will discuss matters related to Kodaira dimensions $\kappa(X,\alpha)$ determined by elements α of the Brauer group $\mathrm{Br}(\mathbb{K})$ of the function field $\mathbb{K}$ of a normal projective variety X.

The main ideas are: (i) such Brauer classes determine log-pairs on X which encode ramification data of maximal orders on X; and (ii) we can study the behaviour of such Kodaira dimensions $\kappa(X,\alpha)$, determined by Brauer classes, as we vary various geometric and algebraic parameters. For instance, we prove invariance of Kodaira dimension for $\mathbb{Q}$-Gorenstein Brauer pairs and also that the Kodaira dimension does not decrease under embeddings of $\mathbb{K}$-central division algebras.

Jack Huizenga (Penn State University)

Negative curves on symmetric blowups of the projective plane and resurgences

The Klein and Wiman configurations of lines are highly symmetric configurations of lines in the projective plane arising from complex reflection groups. One noteworthy property of these configurations is that all the singularities of the configuration have multiplicity at least three. The blowup of the projective plane at the singular points of the configuration is a highly symmetric surface with many interesting properties. We use classical invariant theory to study curves of negative self-intersection on this surface, with the goal of understanding its birational geometry.

The ideal of the singular points in the configuration has many interesting properties from the point of view of commutative algebra. For example, the symbolic cube of this ideal fails to be contained in the ordinary square of the ideal: the product of the lines in the configuration is in the symbolic cube but not the ordinary square. Such a failure of containment demonstrates that theorems of Ein-Lazarsfeld-Smith and Hochster-Hunecke are sharp in the case of the projective plane. Invariants called the resurgence and asymptotic resurgence were introduced by Bocci and Harbourne to systematically study failures of containment of this type. We explain how our study of negative curves allows the computation of these invariants. This is joint work with Thomas Bauer, Sandra Di Rocco, Brian Harbourne, Alexandra Seceleanu, and Tomasz Szemberg.

Patricia Klein (University of Michigan)

Asymptotic Behavior of Certain Koszul Homology Modules

Let (R, m) be a local ring, M a finitely generated module over R, and $f_1, \ldots, f_d$ a system of parameters on M. Lech's limit formula states that

$$\frac{\ell\left(M/(f_1^{t_1}, \ldots, f_d^{t_d})M\right)}{t_1, \cdots, t_d} \to e(f_1, \ldots, f_d|M),$$

the multiplicity of $(f_1, \ldots, f_d)$ on M, as $\min_i t_i \to \infty$. It is natural to ask whether powers of a fixed sequence of parameters may be replaced by any sequence of parameter ideals I_n such that $I_n \subseteq m^n$. Recalling that the multiplicity may be realized as the alternating sum of the lengths of Koszul homology modules, it is also natural to ask for which $i > 0$ we have $\ell\left(H_i(I_n; M)\right)/\ell(M/I_nM) \to 0$. In this talk, we will consider the latter question in the case where R is a complete regular local ring containing a field and M is faithful. We will state some conditions that guarantee the result and give examples of modules and sequences of parameter ideals such that the limit either does not exist or is nonzero.

Kuei-Nuan Lin (Penn State Greater Allegheny)

LCM lattices and dual hypergraphs of square-free monomial ideals.

Given a square-free monomial ideal I in a polynomial ring R over a field k, we would like to know the projective dimension of I. We recall the definition of LCM lattice of a monomial ideal introduced by Gasharov, Peeva and Welker, and the definition of the dual hypergraph of a square-free monomial ideal introduced by Kimura, Terai and Yoshida. We describe the relationship between the LCM lattice and the dual hypergraph of a given square-free monomial ideal. In the joint work with Mantero, we show two square-free monomial ideals have the same projective dimension if they have the same dual hypergraph. We use the properties of LCM lattice to find whether two different dual hypergraphs have the same projective dimension. This is joint work with Sonja Mapes.

Rebecca R.G. (Syracuse University)

Directed families of big Cohen-Macaulay algebras in equal characteristic

A big Cohen-Macaulay algebra over a local ring R is an algebra B such that every system of parameters on R is a regular sequence on B. Previously, Geooffrey Dietz proved that big Cohen-Macaulay algebras in characteristic $p > 0$ form a directed family by studying seed algebras, algebras that have maps to big Cohen-Macaulay algebras. In joint work with Geoffrey Dietz, we extend these results to the equal characteristic 0 case using work of Schoutens on reduction to characteristic p via ultraproducts. I will discuss both sets of results, as well as some applications to closure operations.

Josh Stangle (Syracuse University)

Orders and Non-commutative Crepant Resolutions

In 2004, Van den Bergh defined a non-commutative (crepant) resolution (NCCR) of singularities for a Gorenstein normal domain, R. The definition leads to many strong theorems and connections between commutative algebra and algebraic geometry. Additionally, theorems of Auslander give a constructive analog: NCCRs can be realized as endomorphism rings over R which are maximal Cohen-Macaulay R-modules and have finite global dimension. One goal of current research is to find a definition in the case of Cohen-Macaulay normal domains which replicates some of these strong results and possesses a constructive analog. We will discuss the Gorenstein case and introduce some possible definitions (and obstructions) in the non-Gorenstein case.