Embeddings of flag varieties, and cohomological components in representations of semisimple Lie groups

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Let G and $\widetilde{G}$ be two connected, simply connected, semisimple, finite dimensional, complex Lie groups, with an embedding $\iota : G \hookrightarrow \widetilde{G}$. Upon a suitable choice of Borel subgroups we obtain a G-equivariant embedding of the flag varieties $\iota_o : G/B \longrightarrow \widetilde{G}/\widetilde{B}$. The restriction of homogeneous line bundles results in a G-equivariant pullback:

$$\pi \ : \ H^*(\widetilde{G}/\widetilde{B}, \widetilde{\mathcal{L}}) \longrightarrow H^*(G/B, \iota_o^* \widetilde{\mathcal{L}}).$$

The Borel-Weil-Bott theorem and Schur's lemma imply that π is either surjective or zero. The search for computable conditions for nonvanishing of π reveals interesting relations between branching laws in Representation theory and the Schubert calculus on flag varieties. A necessary and sufficient condition has been found by Dimitrov and Roth, in the case of a diagonal embedding $G \longrightarrow G \times G$, where the pullback can be viewed as a cup product map.

We consider here root embeddings, for which a root system of G is inserted into a root system of $\widetilde{G}$. A suitable formulation of the condition found by Dimitrov and Roth turn out to be valid. The method of proof is substantionally different. It uses a translation of sheaf cohomology on homogeneous varieties to Lie algebra cohomology established by Kostant. There is hope, and some evidences, that this method can be used for other types of embeddings, or to give an alternative proof in the diagonal case.

This work is done under the supervision of Professor Dimitrov.