Frobenius splitting, point counting and degeneration

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Let $\{f = 0\}$ be a degree n hypersurface in affine n-space over $\operatorname{Spec}\mathbb{Z}$. From it, we can construct many other subschemes $\{Y\}$ by decomposing, intersecting, taking unions, and repeating. Then (1) $\operatorname{init} f$ is the product of all the variables implies (2) The number of solutions to $\{f = 0\}$ over $\mathbb{F}_p$ is not a multiple of p implies (3) All the schemes $\{Y\}$ constructed above are reduced over $\mathbb{F}_p$. (If this holds for infinitely many p, e.g. if (1) holds, then reducedness also holds over the rationals.)

Moreover, if (1) holds, then each $\operatorname{init} Y$ is reduced, i.e. Y has a Gröbner basis with squarefree initial ideal. I'll give an example where the set of $\{Y\}$ includes matrix Schubert varieties.